AMPLITUDE RATIOS OF SEVERAL GROUPS OF VARIABLE STARS

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Abstract. The observed positions of classical cepheids, RR Lyrae stars, δ Scuti stars and dwarf cepheids in the $\log g$–$\log T_e$ plane form a continuous sequence, thereby defining the location of maximum instability. The amplitude ratio (the ratio of radial velocity amplitude to light amplitude) is small for variables at the upper end of the instability strip and increases almost linearly towards the lower end of the strip. The theory of radial pulsation predicts the trend of this correlation.

The surface gravities and effective temperatures of variable stars have been obtained by means of matching either the observed absolute energy distribution against the theoretical distribution derived from a model atmosphere (assuming a static atmosphere), or the observed Hγ profile against the theoretical profile. Among the better studied groups of variable stars, classical cepheids, RR Lyrae stars, δ Scuti stars, and dwarf cepheids (or AI Velorum stars), there is strong observational evidence that a direct linear correlation exists between the amplitude of radial velocity variation and amplitude of light variation for each group (Hellerich, 1937; Woltjer, 1956; Kurochkin, 1959; Eggen, 1957; Preston and Paczynski, 1964; Leung and Wehlau, 1967). Much improved information is now available on amplitudes, effective temperatures and surface gravities for these variables. It is of interest to investigate from a more reliable quantitative viewpoint the relationship between these parameters.

Surface gravities and effective temperatures for a number of variable stars have been derived from the methods described. The available data employed for this study are listed as follows: two classical cepheids (η Aql and δ Cep) by Oke (1961a, b); three RR Lyrae stars (RR Lyr, SU Dra, and X Ari) by Oke and Bonsack (1960), Oke *et al.* (1962), and Oke (1966); three δ Scuti stars (ϱ Pup, δ Sct, and δ Del) by Bessell (1969a), three dwarf cepheids (AI Vel, SX Phe and HD 199757) by Bessell (1969b). The mean surface gravity and effective temperature for each group are listed in Table I.

There are three independent sources of observations in the literature for δ Scuti stars: Kuhi and Danziger (1967), Danziger and Dickens (1967) and Bessell (1969a). Bessell (1967) suggested that the differing results of the former authors are most probably due to a systematic underestimation of line-blanketing corrections. The recent work of Breger and Kuhi (1970) also comes to a similar conclusion. In the light of this, we have adopted Bessell's results.

The values of effective temperature and surface gravity (Table I) for these four

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TABLE I

Mean surface gravity, mean effective temperature and amplitude ratio of several groups of variable stars

Variable type	$\log g$	$\log T_e$	R^*_a (km sec^{-1} mag^{-1})	Sources
Classical cepheids	1.3	3.76	33.0 ± 2	Eggen, 1957
RR Lyrae stars	2.5	3.83	48.0 ± 4	Preston and Paczynski, 1964
δ Scuti stars	3.6	3.85	62.5 ± 2	Leung and Wehlau, 1967
Dwarf cepheids	3.8	3.88	76.0 ± 10	this paper

* The amplitudes of light variation are in blue

groups of stars produce a continuous sequence in a theoretical *H–R* diagram. (A sequence for the first three groups was suggested by Danziger and Dickens (1967).) This sequence is located within the theoretical instability boundaries calculated by Böhm-Vitense (1958), Baker and Kippenhahn (1965), Christy (1966a, 1966b), Cox *et al.* (1966), and King *et al.* (1966). Thus it is very likely that all these variables share the same pulsation mechanism.

The relation between the amplitude of radial velocity variation ΔV and the amplitude of light variation Δm, can be expressed by the following linear relation: $\Delta V = R_a \, \Delta m + C$; where C is a constant and is essentially zero in our cases; R_a is the amplitude ratio. The amplitude ratios for classical cepheids, RR Lyrae stars, and δ Scuti stars have been investigated previously and the results and references for them are listed in the last two columns of Table I. Dwarf cepheid variables have been misclassified in the past as δ Scuti stars or RR Lyrae stars. It is for this purpose that the available data on amplitudes of radial velocity variation and light variation for

TABLE II

Amplitudes of radial velocity and of light for dwarf cepheids

Stars	ΔV(km/sec)	Δm(blue mag.)	Sources
SX Phe	50	0.75	Wilson and Walker, 1956
	40	0.50	
	35	0.40	
V 703 Sco	31	0.30	Clube *et al.*, 1969
AI Vel	34	0.40	Herbig, 1949; Walraven, 1955
CY Aqr	88	0.90	Struve, 1949; Hardie and Tolbert, 1961
DY Peg	50	0.70	Bidelman, 1947; Hardie and Geilker, 1958
DY Her	40	0.65	Bonsack, 1957; Hardie and Lott, 1961
HD 199757	~ 30	0.35	Churms and Evans, 1961, 1962

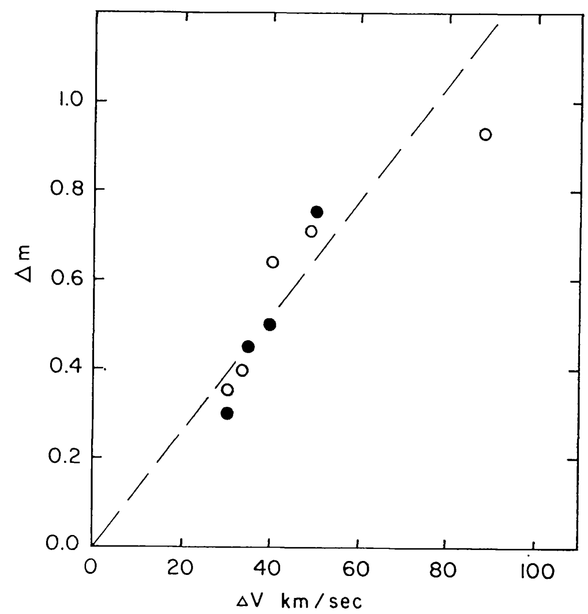

Fig. 1. Light amplitudes (in blue) and radial velocity amplitudes of dwarf cepheids. Filled circles refer to simultaneous observations; open circles refer to non-simultaneous observations.

seven dwarf cepheids are collected in Table II and plotted on Figure 1. The amplitude ratio R_a derived for this group of variables is 76 ± 10 km/sec-mag.

Data from Table I have been plotted on Figures 2 and 3. It is clear from these two diagrams that the amplitude ratio is a function of surface gravity and effective temperature for these stars. That is, in an HR diagram, the amplitude ratio is small for the variables at the upper end of the instability strip and increases almost linearly towards the lower end of the strip. Note that β Canis Majoris stars (Leung, 1968)

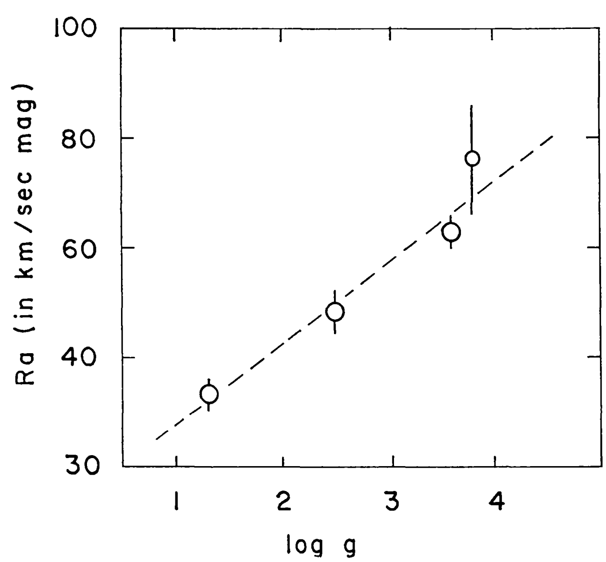

Fig. 2. Surface gravities and amplitude ratios of variable stars.

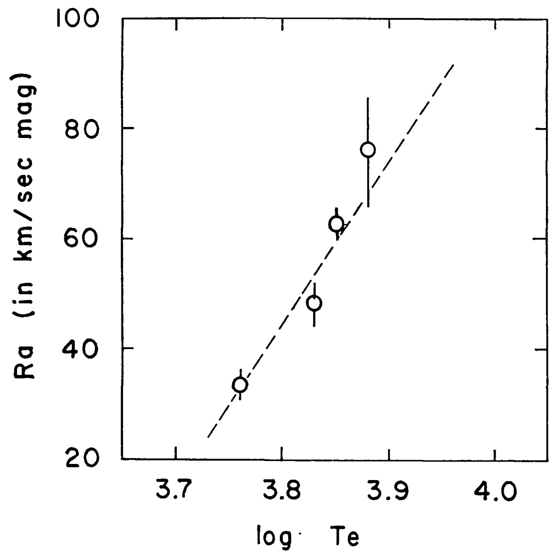

Fig. 3. Effective temperatures and amplitude ratios of variable stars.

do not follow the correlation at all. It is rather common for the variable stars with large surface gravity (δ Scuti stars and dwarf cepheids) to have variable (or modulated) light and velocity curves.

From the linear adiabatic theory of Simon and Stothers (1970, Equation (A-8)) and the non-linear, non-adiabatic theory of Caster (1968, Equation (11)), we find that the amplitude ratio is roughly proportional to radius and inversely to period as expressed by the following equation;

$$R_a \approx K(R/R_{\odot})/P,$$

where K is taken to be constant for stellar envelopes of similar structure, R is radius and P is period. In Table III the quantities R/P and R_a (normalized to unity for the first entry) are listed for the four classes of variable stars in the cepheid instability strip. It is immediately clear that the theory yields a good approximation for the relative values of the amplitude ratio R_a among the different classes of variables.

TABLE III

Comparison of observed amplitude ratios with predicted amplitude ratios from linearized adiabatic pulsation theory

Object	$\langle P^d \rangle$*	$\langle R/R_{\odot} \rangle$*	Normalized R/P	Normalized R_a
Classical cepheids	6.2	60	1.0	1.0
RR Lyr stars	0.63	8	1.3	1.5
δ Sct stars	0.16	3.6	2.3	1.9
Dwarf cepheids	0.078	1.9	2.5	2.3

* For stars in Table I

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